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PROBLEMS POSED AT THE NOVI SAD ALGEBRAIC CONFERENCE '03[1]

Problem 1 *(I. Dolinka) The* random graph *R is the unique (up to isomorphism) countable graph which satisfies the following property: for any two finite disjoint sets A and B of vertices of R there exists a vertex v which is connected by an edge to each vertex of A and no vertex of B. The partial order of principal ideals (i.e., of the $\mathcal{J}$-classes) of the endomorphism monoid End(R) was studied in [1]. The following question remains:*

Is there an uncountable well-ordered chain in the partial order of (principal) ideals of $End(R)$?

References

[1] D. Delić and I. Dolinka, The monoid of the random graph has uncountably many ideals, *Semigroup Forum* **69** (2004) 75–79.

Problem 2 *(M. Erné) If a finite lattice is representable as an interval of topologies (ordered by inclusion), is it representable as an interval of topologies on a finite set?*

References

[1] J. Reinhold, Finite intervals in the lattice of topologies, Papers in honour of Bernhard Banaschewski (Cape Town, 1996.) *Appl. Categ. Structures* **8**, no. 1-2 (2000), 367–376.
[2] M. Erné and J. Reinhold, Lattices of closed quasiorders, *J. Combin. Math. Combin. Comput.* **21** (1996), 41–64.
[3] M. Erné and J. Reinhold, Ordered one-point compactifications, stably continuous frames and tensors. *Quaest. Math.* **22** (1999), 63–81.

Problem 3 *(M. Erné) From a finite lattice* $\mathbf{L}$ *form the* $\mathbf{L}$*-context* $\langle \mathcal{J}, \mathcal{M}, \leq \rangle$, *with* $\mathcal{J}$ *the set of join-irreducible, and* $\mathcal{M}$ *the set of meet-irreducible elements. Then build the concept lattice of the complementary context* $\langle \mathcal{J}, \mathcal{M}, \not\leq \rangle$. *This gives a kind of negation* $\mathcal{C}\mathbf{L}$. *It is known that the sequence* $(\mathcal{C}^n\mathbf{L})_{n\in\omega}$ *ends with a self-negated lattice* $\mathbf{N}$, *i.e.* $\mathbf{N} \cong \mathcal{C}\mathbf{N}$, *or a pair of mutual negations. Is there a short, intrinsic characterization of self-negated lattices?*

References

[1] K. Deiters and M. Erné, Negations and contarpositions of complete lattices, *Discrete Math.* **181** (1998), 91–111.

[1] Collected by **N. Mudrinski**, prepared for publication by **P. Marković**.

Problem 4 *(M. Goldstern) Let* **L** *be a lattice. We will say it is* 1-order polynomially complete *if for all f, $f : L \to L$ is a monotone map, then f is a polynomial function.*
Are there infinite 1-order polynomially complete lattices?

Background: A lattice **L** is *order polynomially complete* when every monotone function $f : L^n \to L$ is a polynomial function.

References

[1] M. Goldstern and S. Shelah, Order-polynomially complete lattices must be LARGE, *Algebra Universalis* **39** (1998), 197–209.
[2] M. Goldstern and S. Shelah, There are no order-polynomially complete lattices, after all, *Algebra Universalis* **42** (1999), 49–57.
[3] M. Goldstern, Unary opc, To appear in *Contributions to General Algebra* **16**, (eds. Dorfer, Pöschel, Chajda, Halas, Eigenthaler, Müller).

Problem 5 *(L. Kwuida) A weakly dicomplemented lattice is an algebra $\langle L; \wedge, \vee, {}^{\triangle}, {}^{\nabla}, 0, 1\rangle$ of type $(2, 2, 1, 1, 0, 0)$ such that $\langle L; \wedge, \vee, 0, 1\rangle$ is a bounded lattice and the unary operations satisfy*

1 $x^{\triangle\triangle} \leq x$ **1'** $x \leq x^{\nabla\nabla}$

2 $x \leq y \Rightarrow x^{\triangle} \geq y^{\triangle}$ **2'** $x \leq y \Rightarrow x^{\nabla} \geq y^{\nabla}$

3 $(x \wedge y) \vee (x \wedge y^{\triangle}) = x$ **3'** $(x \vee y) \wedge (x \vee y^{\nabla}) = x$

This structure arises in Contextual Logic. A primary filter of a weakly dicomplemented lattice is a proper filter containing x or $x^{\triangle}$ for every element x. The notion of primary ideal is dually defined. The "Prime Ideal Theorem" holds, namely, if F is a filter which does not intersect an ideal I, then there is a primary filter G containing F such that $G \cap I = \emptyset$. This is unfortunately not enough to get a representation theorem. So we pose the following problem.

Let I be an ideal of a weakly dicomplemented lattice and $x \notin I$. Is there any primary filter G such that $x \notin G$ and $G \cap I = \emptyset$?

Problem 6 *(L. Kwuida) The skeleton of a weakly dicomplemented lattice L is the*

$$S(L) := \{x \in L \mid x^{\nabla\nabla} = x\}.$$

From axioms 2' and 3' it follows that

$$(x \vee x^{\nabla}) \wedge (x \vee x^{\nabla\nabla}) = x \quad \text{and } (x \vee x^{\nabla})^{\nabla} = 0.$$

The skeleton $S(L)$ is an ortholattice (cf. [4]). The idea is to describe congruences of weakly dicomplemented lattices by mean of congruences of their skeleton (cf. [1]). Is there any description of ortholattice congruences?

More details can be found in [2] and [3].

REFERENCES

[1] T. Katriňák, The structure of distributive double p-algebras. Regularity and congruences, *Algebra Universalis* **3** (1973), 238-246.
[2] R. Wille, Boolean Concept Logic, Proceedings of the Linguistic on Conceptual Structures: Logical Linguistic, and Computational Issues, 317-331, *Lecture Notes in Computer Science* **1867** (2000), Springer-Verlag.
[3] L. Kwuida, Dicomplemented lattices. A Contextual generalization of Boolean algebras (2004), Shaker Verlag.
[4] L. Kwuida, A. Tepavčević, B. Šešelja, Negation in contextual logic, Conceptual Structures at Work: 12th International Conference on Conceptual Structures, Proceedings, 227–241 em Lecture Notes in Computer Science **3127** (2004), Springer-Verlag.

Problem 7 *(P. Marković) This is the problem of P. Frankl, posed in 1979. The reference list on this famous question is too long to cite, but it still remains open today.*

Given a finite family $\mathcal{F}$ of finite sets, closed under taking unions, $\mathcal{F} \neq \{\emptyset\}$, does there always exist an element $a \in \bigcup \mathcal{F}$ such that a is an element of at least half of the sets in $\mathcal{F}$?

An equivalent, alternative statement of this problem is:

Given a finite lattice $\mathbf{L}$, does there always exist a meet-irreducible element $a \in L$ such that $|a\downarrow| \leq 0.5|L|$?

Problem 8 *(Péter Pál Pálfy) Is there a minimal clone that contains infinitely many binary operations?*

Background: A clone of operations is minimal if it is generated by each nontrivial operation in this clone. (The trivial operations are the projections.) An essentially minimal clone with infinitely many binary operations has been constructed by H. Machida and I. Rosenberg [1].

REFERENCES

[1] H. Machida and I. G. Rosenberg, A "large" essentially minimal clone over an infinite set, Proc. Int. Conf. on Algebra (Novosibirsk, 1989), Part 3, 159–167, *Contemporary Math.* **131**, Amer. Math. Soc., Providence, RI, 1992.

Problem 9 *(M. Ploščica) Let* $\mathbf{L}$ *be a distributive algebraic lattice. The set* $F = \{x \in L \,|\, 1 \text{ is compact in } \uparrow x\}$ *is a filter. Let us define a lattice* $\mathbf{L}' \leq \mathbf{L} \times \mathbf{2}$ *with the universe* $L' = \{\langle x, i\rangle \,|\, i = 0 \text{ or } x \in F\}$. *Must* $\mathbf{L}'$ *be algebraic?*

Remark: The problem is connected with the relationship between congruence lattices of bounded and unbounded lattices. The topological version of this problem is as follows: Let S be a space having a basis of compact open sets. Let S' be the one-point compactification of S. Does S' have a basis of compact open sets?

Problem 10 *(M. Ploščica) Let* $\mathcal{V}$ *be a finitely generated, congruence distributive variety. Is it true that for an infinite* $\mathbf{A} \in \mathcal{V}$, *the number of compact elements of the lattice* $\mathbf{Con}(\mathbf{A})$ *is equal to* $|A|$*?*

Remark: The conjecture is easily seen to be true for $|A| = \aleph_0$.

Problem 11 *(C. Szabó and V. Vértesi) Consider the following algorithmic question:*
Input: *Two group terms, t_1 and t_2.*
Question: *Are they equal over the symmetric group $\mathbf{S}_4$, namely do they agree at every substitution of elements for variables?*
What is the computational complexity of the above question?

Background: Questions of this kind are always in coNP. For the nonsolvable groups the question is coNP-complete, while for nilpotent groups and for some metacyclic group, particularly $\mathbf{A}_4$, it is in P.